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A SIMPLE METHOD FOR TREATING DIFFRACTIVE PROPAGATION FROM A LAS--ETC(U)  
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TECHNICAL REPORT RH-80-2

**A SIMPLE METHOD FOR TREATING DIFFRACTIVE  
PROPAGATION FROM A LASER WITH DIAGNOSTIC  
OPTICS**

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19 November 1979



**U.S. ARMY MISSILE COMMAND**

*Redstone Arsenal, Alabama 35809*

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In studying mode-formative properties of lasers and in other connections we have had occasion to manipulate the output of a laser by spherical mirrors or other optical elements prior to observation of an intensity distribution. Provided there is no further aperturing of the beam beyond the laser output, the propagation can be described by a Fresnel integral with an appropriate value of effective propagation distance in free space. To our knowledge, no physically transparent method for determining the value of this propagation distance has been presented for the general case, though Collins' ray-optics diffraction scheme [1] encompasses it provided one works through all the matrix manipulations in detail, and a transformation by Sziklas and Siegman [2,3] can be rather readily applied to the special case of one lens and an observation point preceding its focal point. We refer explicitly only to lenses, but with the understanding that the equivalent mirrors are considered to be included. The paraxial approximation is assumed to hold everywhere.

From physical plausibility arguments one could expect to determine the effective propagation distance by removing one lens at a time, starting with the last one, and moving from the observation (or image) plane to its conjugate (object) plane. While this prescription has physical appeal, its justification requires more than the plausibility argument which mixes a concept (imaging) normally used in connection with incoherent radiation with a concept (diffraction) which is meaningful only for coherent radiation. A detailed justification of the prescription will be given in the following paragraphs. The scheme has also been experimentally confirmed, as will be discussed.

For definiteness, the scheme is depicted in Figure 1 (a) for the particularly simple case of a single lens inserted into the collimated output beam of a laser some distance downbeam from the output. The plane at which the lateral intensity distribution is to be observed is labeled I (following the notion of the image location). For the present discussion we ignore portions of the figure to the right of the image plane. The location of the plane labeled O is determined by the Gaussian lens formula

$$1/d_0 + 1/d_I = 1/f \quad (1)$$

The prescription then is that  $d_{eff}$  is simply the distance from the source to the plane O, as illustrated. In this case we assume  $d_{eff}$  to be the same as if it were in front of the source, i.e.,

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$$d_{\text{eff}} = d_1 - d_0 \quad (2)$$

Note that 0 may be behind the source, i.e.,  $d_{\text{eff}}$  may be negative; this is equivalent to a problem with positive  $d_{\text{eff}}$  but with an inverted phase distribution at the source aperture. The effective propagation distance  $d_{\text{eff}}$  can be used in conjunction with the output aperture halfwidth,  $a$ , and the wavelength,  $\lambda$ , to form an effective Fresnel number

$$F_{\text{eff}} = a^2 / \lambda d_{\text{eff}} \quad (3)$$

The smaller the value of  $F_{\text{eff}}$ , the nearer the problem has approached the Fraunhofer limit, in which the observed intensity is the absolute square of the Fourier transform of the complex wave amplitude of the laser output.  $F_{\text{eff}}$  is closely related to the collimated beam Fresnel number which has been defined by Siegman [2] in connection with unstable resonators. Propagation of a collimated output wave from the laser through a distance  $d_{\text{eff}}$  would result in an intensity distribution which, except for a transverse scaling factor, is the same as the actual observed distribution.

We follow the notation of Siegman [4] (slightly different from that of Collins) in defining the ABCD matrix of paraxial optics such that the position  $r$  and slope  $r'$  of a ray after passing through an optical system is given in terms of the position  $r_0$  and slope  $r'_0$  of the corresponding input ray by the matrix relation

$$\begin{pmatrix} r \\ r' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_0 \\ r'_0 \end{pmatrix} \quad (4)$$

The overall ABCD matrix of the optical system is, of course, the matrix product, in reverse order of the ABCD matrices for the various individual components of the system. For simplicity of discussion, the Collins [1] diffraction integral in one transverse dimension can be written in the same form as given in a paper by Siegman [4].

$$E(x) = \left( \frac{1}{B\lambda} \right)^{1/2} \int_{-\infty}^{\infty} E_0(x_0) \exp \left\{ -\frac{j\pi}{B\lambda} (Ax_0^2 - 2xx_0 + Dx^2) \right\} dx_0. \quad (5)$$

Here  $E_0(x_0)$  and  $E(x)$  are the complex input and output wave amplitudes, and a factor of unit modulus has been suppressed. The effective propagation distance is simple  $B/A$ , as can be readily seen by comparison of the previous equation with the form of a conventional Fresnel integral.

To justify the above prescription, first consider the given overall optical system with the final lens removed, using an observation plane which is the conjugate of the final observation plane with respect to the final lens. The effective propagation distance for the reduced (one lens removed) system is taken to be  $B/A$ , and we seek to determine the effective propagation distance  $B''/A''$  of the overall system. The overall transfer matrix  $M''$  will be given by  $M'M$ , where  $M$  is the ABCD matrix of the reduced system, and  $M'$  is the matrix describing propagation by a distance  $d_0$  to the final lens, the effect of the final lens, and further propagation by a distance  $d_1$  to the final observation plane. For brevity the steps involved in the derivation are outlined. Because of the conjugate relationship of the two observation planes it follows that  $B' = 0$ . This in turn establishes that

$$A'' = A' A, \quad (6)$$

$$B'' = A' B. \quad (7)$$

Now by division, one readily obtains that the effective propagation distance  $B''/A''$  of the overall system is simply  $B/A$ , which is of course the propagation distance for the reduced system as referred to previously. It can also be shown that the lateral magnification is what one would expect on physical plausibility grounds, i.e.,  $-d_1/d_0$ .

The test configuration for measurement of Fresnel/Fraunhofer patterns is shown in Figure 1. A helium neon laser with spatial filter is used as a clean, coherent source of diverging spherical waves of light. An achromatic lens, corrected for spherical aberration is positioned to provide a collimated beam. In Figure 1 (c), a small rectangular aperture (0.635 cm x 1.27 cm) masking most of the beam is placed immediately behind the lens. This apertured, planar wavefront beam propagates a distance  $d_{eff}$ , the Fresnel pattern in this plane being magnified and imaged upon a camera film plane by a lens of short focal length.

The intensity distributions of Figure 1(c) are directly correlated (except for magnification or inversion) with the



patterns of Figures 1(a) and 1(b) with a 74.37-cm focal length telescopic objective lens in place. The objective lens is placed approximately 4400 cm from the rectangular aperture in Figure 1(a) and against the aperture in Figure 1(b). The intensity distributions are obtained at an arbitrary distance  $d_I$  behind the telescopic objective lens. The image of this intensity distribution is magnified by a short focal length lens and imaged on a camera film plane.

Using the Gaussian lens formula one would expect the pattern of Figure 1(c) to be related to those of Figures 1(a) and 1(b) by the following equation:

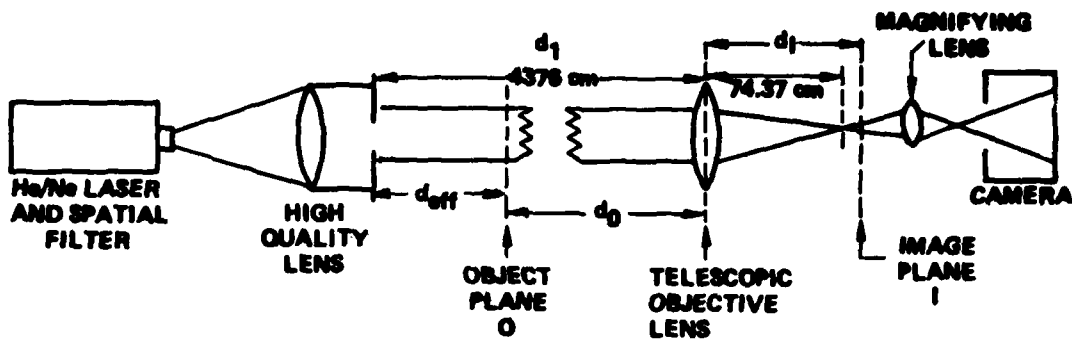
$$d_{eff} = \left| d_I - (d_I f / d_I - f) \right| . \quad (8)$$

A verification that the Fresnel pattern obtained in Figure 1(c) is analogous to a Fresnel pattern in Figures 1(a) and 1(b) is shown in the following photographs. Figure 2(a) was taken a distance of 4376 cm from the rectangular aperture of Figure 1(c). Accordingly, an image plane  $d_I$  of 75.007 cm was chosen in Figure 1(a) to give an object plane a distance 8752 cm to the left of the telescopic objective lens ( $d_0 - d_I = 4376$  cm). This intensity distribution is shown in Figure 2(b). The intensity distribution is essentially identical to that of Figure 2(a) which one would expect for a plane wave incident on an aperture.

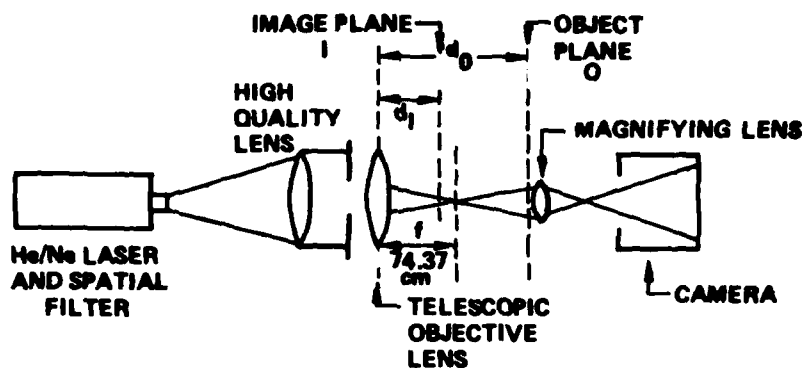
Another experimental verification was obtained with the rectangular aperture against the lens as in Figure 1(b). In this case, image planes both inside and outside of focus may be chosen to be equivalent to an image plane in Figure 1(c). Figure 3(a) is a Fresnel pattern obtained with  $d_{eff}$  being 93 cm from the rectangular aperture of Figure 1(c). An image plane in Figure 2(c) of 371.37 cm ( $d_0 = 93$  cm) is shown in Figure 3(b). This is equivalent to an image plane of  $d_I = 41.3$  cm inside of focus ( $d_0 = -93$  cm) shown in Figure 3(c).

The conclusion obtained from these and other photographs indicates the validity of using the Gaussian lens formula to relate free space propagation of a laser beam to that obtained in an imaging system. In the case described previously, a plane parallel wavefront was vignetted by a rectangular aperture and allowed to propagate a given distance. In practice, any wavefront, e.g., the complex beam from an unstable resonator, can be treated in the

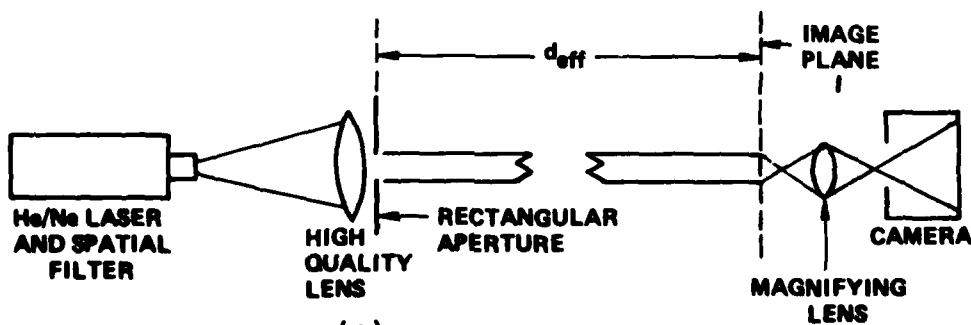
same manner provided the imaging system is of sufficient size to intercept the bulk of all spatially diffracted components of light.



(a)

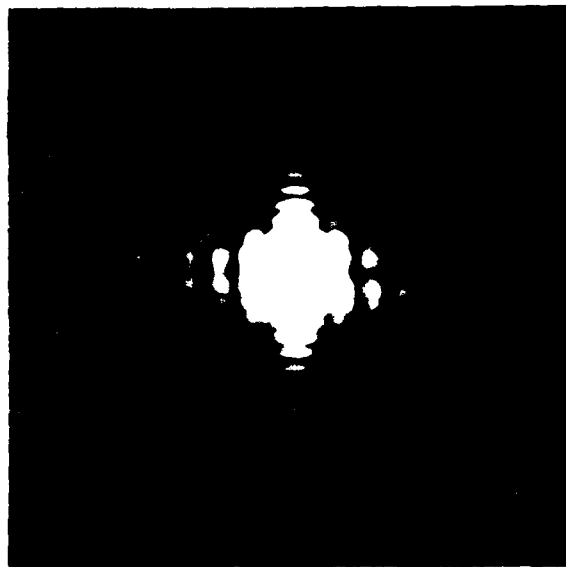


(b)

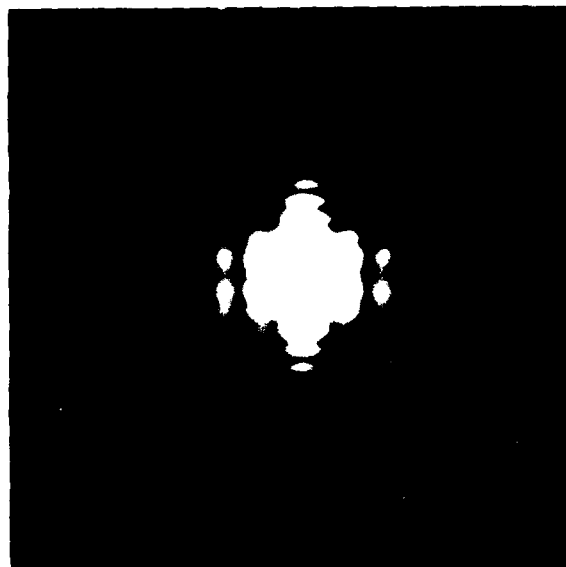


(c)

Figure 1. Experimental configurations for determining equivalency of Fresnel/Fraunhofer distributions.

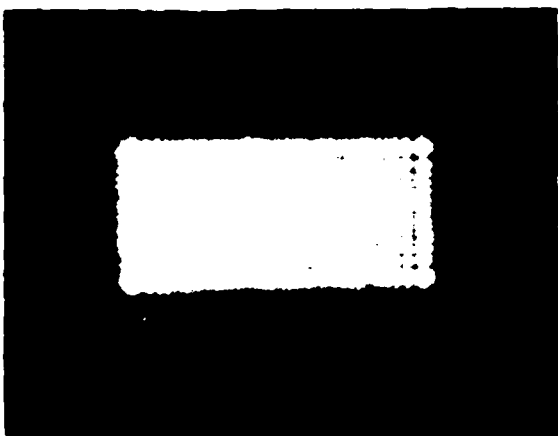


(a)

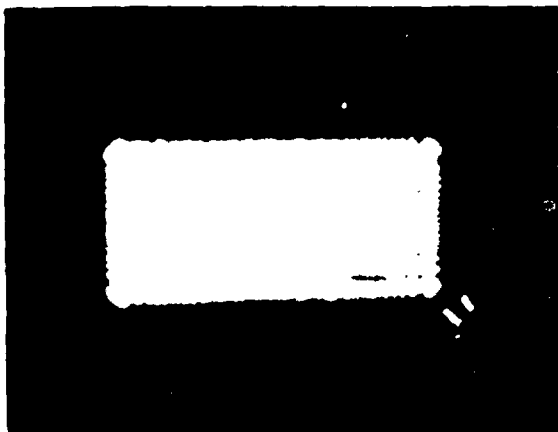


(b)

Figure 2. Diffractive propagation of a rectangular laser beam over an effective distance,  $d_{eff}$ , of 4376 cm.



(a)



(b)



(c)

Figure 3. Diffractive propagation of a rectangular laser beam over an effective distance,  $d_{\text{eff}}$ , of 93 cm.

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